

**Electricity and New Energy**

# **Digital Servo Motor Control**

**Courseware Sample**

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By the staff of Festo Didactic

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




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






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# Safety and Common Symbols

The following safety and common symbols may be used in this manual and on the equipment:

Symbol	Description
	<b>DANGER</b> indicates a hazard with a high level of risk which, if not avoided, will result in death or serious injury.
	<b>WARNING</b> indicates a hazard with a medium level of risk which, if not avoided, could result in death or serious injury.
	<b>CAUTION</b> indicates a hazard with a low level of risk which, if not avoided, could result in minor or moderate injury.
	<b>CAUTION</b> used without the <i>Caution, risk of danger</i> sign  , indicates a hazard with a potentially hazardous situation which, if not avoided, may result in property damage.
	Caution, risk of electric shock
	Caution, hot surface
	Caution, risk of danger
	Caution, lifting hazard
	Caution, hand entanglement hazard
	Notice, non-ionizing radiation
	Direct current
	Alternating current
	Both direct and alternating current
	Three-phase alternating current
	Earth (ground) terminal

# Safety and Common Symbols

Symbol	Description
	Protective conductor terminal
	Frame or chassis terminal
	Equipotentiality
	On (supply)
	Off (supply)
	Equipment protected throughout by double insulation or reinforced insulation
	In position of a bi-stable push control
	Out position of a bi-stable push control

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# Preface

A servomechanism is an automatic device that uses error-sensing feedback to correct the error in the mechanism. A servo motor, which is a type of servomechanism, is provided with a sensor (e.g., an incremental encoder, a position potentiometer, a speed sensor) that compares the command (e.g., the applied voltage) with the actual movement (e.g., the motor speed). Using a controller and appropriate control strategies, the error existing between the command and the actual movement can be determined, analyzed, and then corrected.

Servo motors are used more and more because they give much more precision and/or rapidity to the movements of a mechanical system. An industrial robot, for example, usually contains many servo motors.

The Digital Servo Training System is a compact trainer designed to familiarize students with the fundamentals of digital servo control. The system features a single-axis, belt-driven positioning system, a digital servo controller, and powerful software tools. Control of the motor can be achieved in several ways: using the included hardware controller, LABVIEW or MATLAB/SIMULINK, or an optional analog controller.

Open-source firmware and software controls are provided so the user can create his own control strategies by modifying the existing ones or by developing new ones. This open architecture also facilitates the addition of mechanical options to the system.

The present manual, Digital Servo Motor Control, familiarizes students with the internal characteristics of a servo motor. It also allows students to experiment with different types of control loops and expand their knowledge of servo control.

We invite readers of this manual to send us their tips, feedback, and suggestions for improving the book.

Please send these to [did@de.festo.com](mailto:did@de.festo.com).

The authors and Festo Didactic look forward to your comments.



# To the Instructor

You will find in this Instructor Guide all the elements included in the Student Manual together with the answers to all questions, results of measurements, graphs, explanations, suggestions, and, in some cases, instructions to help you guide the students through their learning process. All the information that applies to you is placed between markers and appears in red.

## **Accuracy of measurements**

The numerical results of the hands-on exercises may differ from one student to another. For this reason, the results and answers given in this manual should be considered as a guide. Students who correctly performed the exercises should expect to demonstrate the principles involved and make observations and measurements similar to those given as answers.



**Sample Exercise**  
**Extracted from**  
**the Student Manual**  
**and the Instructor Guide**



## Servo Closed Loop Speed Control – Steady State Characteristics

### EXERCISE OBJECTIVE

When you have completed this exercise, you will be familiar with servo operation in closed loop speed control. You will know how to calculate and measure the steady state speed of the Digital Servo in closed loop speed control for various controller gains both theoretically and experimentally and be able to compare the two.

### DISCUSSION OUTLINE

The Discussion of this exercise covers the following points:

- Components of the Digital Servo operating under closed loop speed control
- Sensor and power amplifier gain

### DISCUSSION

#### Components of the Digital Servo operating under closed loop speed control

The Digital Servo closed loop speed-control system consists of the following.

- A dc brush-type servo motor
- A speed sensor, i.e., an incremental encoder directly coupled to the motor shaft
- A system controller
- A human machine interface (HMI) used for setting the controller parameters, function generator, and recorder functions

For brevity purposes, we will now refer to the motor steady state speed constant  $K_S$  as the general motor speed constant  $K$ .

Figure 23 shows the simplified block diagram of a servo motor closed loop speed-control system with a first-order model (developed in Exercise 2). The controller is proportional only, which means that it has a constant gain term (proportional action is discussed in more detail in Exercise 9).

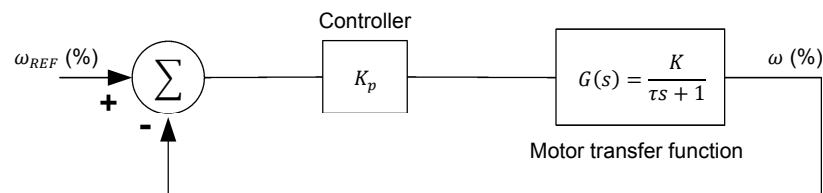


Figure 23. Block diagram of a servo motor in closed loop speed-control mode.

The controller gain  $K_p$  is the result of the **PID controller** three different gains: the proportional gain  $K_p$ , the integral gain  $K_i$  and the derivative gain  $K_d$ . In most

exercises in this manual, the controller gain is equivalent to the proportional gain  $K_p$ , as only proportional action is present in the system.

Analysis of the block diagram components in Figure 23 (see Appendix C for a detailed analysis), shows that the steady state speed  $\omega_{SS}$  of the closed loop system can be calculated as follows:

$$\omega_{SS} = \left( \frac{K_c K}{1 + K_c K} \right) \omega_{REF} \quad (26)$$

where

$\omega_{SS}$	is the motor steady state speed (controlled or process variable)
$\omega_{REF}$	is the desired or reference motor speed (set point)
$K$	is the motor speed constant [(rad/s)/V]
$K_c$	is the controller gain (adjustable)

Equation (26) shows that, as the controller gain (proportional only)  $K_c$  increases, the  $(K_c K)/(1 + K_c K)$  ratio approaches 1. In other words, the higher the gain, the more the motor steady state speed  $\omega_{SS}$  approaches the desired or reference speed  $\omega_{REF}$ . The difference between the reference speed and the actual speed ( $\omega_{REF} - \omega$ ) is referred to as the error (or offset). Therefore, increasing the proportional gain decreases the error. This means that, theoretically, the proportional gain could be set to a very high value in order to minimize the error. In practice, however, increasing the proportional gain destabilizes the servo system and produces speed changes and oscillations. This is discussed in more details in Exercise 5.

### Sensor and power amplifier gain

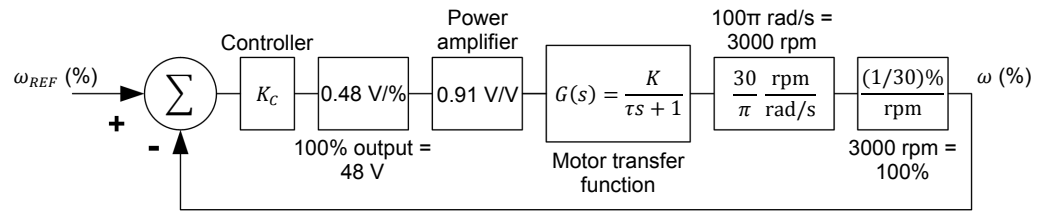
In a real servo system implementation, the analysis must consider the gains of both the servo system power amplifier and the speed sensor. The reference speed  $\omega_{REF}$  for the servo system as well as the controlled variable speed  $\omega_{SS}$  is often expressed in percentage, as is the case for the Digital Servo controller. The conversion between percentage and speed must be taken into account and can be seen as another gain term. All gain terms can then be grouped as one single term by multiplying them together.

The following gain terms are determined for the Digital Servo:

- A 100% output from the controller output is equivalent to 48 V. The gain for converting percentage output is thus 0.48 V/%.
- The power amplifier gain is of approximately 0.91. A 100% output from the controller thus results in only 0.91 x 48 V being applied to the dc motor. The power amplifier gain is due to the motor output electronic design. This means that it cannot output more than 91% of its entry value of 48 V dc.
- The conversion of rad/s to rpm can be represented as a gain term of  $\frac{30}{\pi}$  rpm/(rad/s).
- A 100% speed is equal for the Digital Servo to 3000 rpm. The gain term for converting rpm to percentage is thus (1/30)%/rpm.

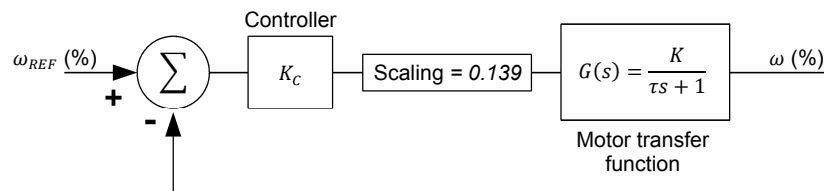


The block diagram in Figure 24 shows all of these gain terms:



**Figure 24.** Block diagram of a servo motor in closed loop speed-control mode showing all gain terms.

All the gain terms in Figure 24 can be grouped together as a total product of all terms. In this case, the product is 0.139 V/(rad/s) (0.48 x 0.91 x 30/π x 1/30). This gain will be referred to as a scaling factor. A block diagram that shows the grouping of these gains is given in Figure 25.



**Figure 25.** Block diagram of a servo motor in closed loop speed-control mode showing the simplified gain term.

From the above, it can be seen that for this particular system, Equation (26) has to be modified to the following:

$$\omega_{SS} = \left( \frac{0.139K_C K}{1 + 0.139K_C K} \right) \omega_{REF} \quad (27)$$

## PROCEDURE OUTLINE

The Procedure is divided into the following sections:

- Setup and connections
- Closed loop speed-control measurements

## PROCEDURE

### Setup and connections

*In this section, you will setup the Digital Servo for closed-loop speed-control measurements.*

#### 1. Make the following settings on the Digital Servo system:

- Setup the servo system for speed control, i.e., disengage the platform.

- Set the belt tension to allow the belt to be lifted of the pulley connected to the motor shaft and slipped on the two pins to the rear of the pulley, allowing the shaft to run uncoupled from the belt.
- Secure the flywheel to the shaft using the appropriate hex key.

2. Run LVServo, and click on the **Device Controlled** button in the **Speed Loop** menu. Make sure the settings are initially as shown in Table 12:

Table 12. Settings for closed loop speed-control measurements.

Function Generator		Trend Recorder	
Signal Type	<i>Constant</i>	Reference	<i>Checked</i>
Frequency	<i>1 Hz</i>	Speed	<i>Checked</i>
Amplitude	<i>0%</i>	Current	<i>Unchecked</i>
Offset	<i>0%</i>	Voltage	<i>Checked</i>
Power	<i>Off</i>	Error	<i>Unchecked</i>
PID Controller		$K_p \times \text{Error}$	<i>Unchecked</i>
Gain ( $K_p$ )	<i>1</i>	Error Sum / $t_i$	<i>Unchecked</i>
Integral Time ( $t_i$ )	<i>Inf (Off)</i>	$t_d \times \text{Delta Error}$	<i>Unchecked</i>
Derivative Time on E ( $t_d$ (E))	<i>0</i>	PID Output	<i>Unchecked</i>
Derivative Time on PV ( $t_d$ (PV))	<i>0</i>	Display Type	<i>Sweep</i>
Timebase	<i>10 ms</i>	Show and Record Data	<i>On</i>
Anti-Reset Windup	<i>On</i>	Measured Gain (rpm)	<i>3000</i>
Upper Limit	<i>100%</i>	Measured Gain (A)	<i>7</i>
Lower Limit	<i>-100%</i>	Measured Gain (V)	<i>48</i>
Open or Closed Loop	<i>Closed</i>		
PV Speed Scaling			
100% Value	<i>3000 rpm</i>		

3. Set the function generator **Power** switch to ON.

### Closed loop speed-control measurements

In this section, you will calculate the steady state speed  $\omega_{SS}$  and the speed constant  $K$  value of the motor operating under closed loop speed control for various gains using Equation (27). You will then measure experimentally the motor  $\omega_{SS}$  and  $K$  values for various gains and compare the theoretical and experimental results. You will eliminate the calculated error value by means of integral action. Only proportional action will be used. The controller gain value is thus equal to the proportional gain value and will be referred to as  $K_p$ .

- Slowly increase the offset value until the motor voltage reading reaches 40%, which corresponds to a voltage of 19.2 V. Record the actual speed  $\omega$  (rpm) and the reference speed  $\omega_{REF}$  (%) in Table 13:

Table 13. Speed readings during closed loop speed-control measurements.

Gain	Voltage		Reference Speed		Actual Speed		Error	Speed/Voltage Ratio		Steady State Speed Ratio	Steady State Speed
	$E$	$E$	$\omega_{REF}$	$\omega_{REF}$	$\omega$	$\omega$		$\Delta\omega = \omega_{REF} - \omega$	$K = \omega/E$		
$K_C$	%	V	%	rad/s	rpm	rad/s	%	%	(rad/s)/V	$\omega_{SS}/\omega_{REF}$	%
1	40	19.2	81	254.5	1115	116.8	37.17	43.83	6.0814	0.4581	37.10
2	40	19.2	59	185.4	1115	116.8	37.17	21.83	6.0814	0.6283	37.07
3	40	19.2	52	163.4	1120	117.3	37.33	14.67	6.1087	0.7181	37.34
4	40	19.2	48	150.8	1114	116.7	37.13	10.87	6.0759	0.7716	37.04
5	40	19.2	46	144.5	1118	117.1	37.27	8.73	6.0977	0.8091	37.22

The following table shows the different measured parameters:

Speed readings during closed loop speed-control measurements.

Gain	Voltage		Reference Speed		Actual Speed		Error	Speed/Voltage Ratio		Steady State Speed Ratio	Steady State Speed
	$K_C$	$E$	$E$	$\omega_{REF}$	$\omega_{REF}$	$\omega$		$\omega$	$\omega$		
	%	V	%	rad/s	rpm	rad/s	%	%	(rad/s)/V		%
1	40	19.2	81	254.5	1115	116.8	37.17	43.83	6.0814	0.4581	37.10
2	40	19.2	59	185.4	1115	116.8	37.17	21.83	6.0814	0.6283	37.07
3	40	19.2	52	163.4	1120	117.3	37.33	14.67	6.1087	0.7181	37.34
4	40	19.2	48	150.8	1114	116.7	37.13	10.87	6.0759	0.7716	37.04
5	40	19.2	46	144.5	1118	117.1	37.27	8.73	6.0977	0.8091	37.22

5. Decrease the offset to 0%, increase the proportional gain  $K_p$  to 2 and repeat the previous operation. Do the same thing for  $K_p$  values of 3, 4, and 5.
6. Fill out the rest of Table 13 using Table 14 as a quick reference for speed unit conversion. Keep in mind the following while completing Table 13:
  - $K$  is the ratio of speed  $\omega$  to supply voltage  $E$  and is calculated by dividing the speed value in rad/s by the supply voltage (V).
  - The error value is calculated by subtracting the speed  $\omega$  value to the reference speed  $\omega_{REF}$ .
  - The steady state speed ratio  $\omega_{SS}/\omega_{REF}$  and speed value  $\omega_{SS}$  (%) are calculated using Equation (27).

Table 14. Speed unit conversion quick reference.

Speed unit type	Multiply by
rpm → rad/s	$\frac{\pi \text{ rad/s}}{30 \text{ rpm}}$
rad/s → rpm	$\frac{30 \text{ rpm}}{\pi \text{ rad/s}}$
% → rad/s	$\pi \text{ rad/s}$
rad/s → %	$\frac{1}{\pi \text{ rad/s}}$

7. Compare the calculated steady state speed with the measured steady state speed.

The calculated and measured steady state speed values are very similar.

8. Describe what happens to the error as the proportional gain  $K_p$  value increases.

The error value decreases as the proportional gain  $K_p$  value increases.

9. Set the proportional gain  $K_p$  back to 1 and enter 0.1 s into the integral time  $t_i$ . Describe what happens to the error when integral action is introduced into the controller.

When integral action is introduced in the controller, the error value is eliminated.

## CONCLUSION

In this exercise, you familiarized yourself with servo system operation in closed loop speed control. You learned how to calculate and measure the steady state speed of the Digital Servo in closed loop speed control. You also learned to calculate the error value between the reference speed and the actual speed and how to minimize it by increasing the controller gain.

**REVIEW QUESTIONS**

1. Consider a dc motor system having a supply voltage  $E$  of 35 V. The dc motor system proportional gain  $K_p$  is set to 3 and its steady state motor speed  $\omega_{SS}$  is 1960 rpm. Find the motor system speed constant  $K$  (steady state speed to voltage ratio):

The motor system speed constant  $K$  is calculated so:

$$\omega_{SS} = 1960 \text{ rpm} \times \frac{\pi \text{ rad/s}}{30 \text{ rpm}} = 205.3 \text{ rad/s}$$

$$K = \frac{205.3 \text{ rad/s}}{35 \text{ V}} = 5.87 \text{ rad/s/V}$$

2. Given the same motor parameters as in question 1, find the motor reference speed  $\omega_{REF}$  in rad/s, rpm, and percentage, as well as the steady state closed loop system value ( $\omega_{SS}/\omega_{REF}$ ).

$$\omega_{SS} = 205.3$$

$$\frac{0.139K_p K}{1 + 0.139K_p K} = 0.71$$

$$\omega_{REF} = \frac{205.3 \text{ rad/s}}{0.71} = 289.2 \text{ rad/s}$$

$$\omega_{REF} = 289.2 \text{ rad/s} \times \frac{30 \text{ rpm}}{\pi \text{ rad/s}} = 2761.3 \text{ rpm}$$

$$\omega_{REF} = \frac{2761.3 \text{ rpm}}{3000 \text{ rpm}} \times 100\% = 92\%$$

$$\frac{\omega_{SS}}{\omega_{REF}} = \frac{1960 \text{ rpm}}{2761.3 \text{ rpm}} = 0.71$$

3. Given the same motor parameters as in question 1, find the system error value in percentage.

The error is equal to:

$$\text{Error} = 92.0\% - \frac{1960 \text{ rpm}}{3000 \text{ rpm}} \times 100\% = 26.7\%$$

4. Given the same motor parameters as in question 1, calculate what happens to the reference speed  $\omega_{REF}$  (rad/s, rpm, and percentage) when the proportional gain  $K_p$  value is set to 4, as well as the resulting steady state closed loop system value ( $\omega_{SS}/\omega_{REF}$ ).

$$\omega = 205.3 \text{ rad/s}$$

$$K = 5.87 \text{ rad/s}$$

$$\frac{0.139K_p K}{1 + 0.139K_p K} = 0.765$$

$$\omega_{REF} = \frac{205.3 \text{ rad/s}}{0.765} = 268.2 \text{ rad/s}$$

$$\omega_{REF} = 268.2 \text{ rad/s} \times \frac{30 \text{ rpm}}{\pi \text{ rad/s}} = 2561.5 \text{ rpm}$$

$$\omega_{REF} = \frac{2561.5 \text{ rpm}}{3000 \text{ rpm}} \times 100\% = 85.4\%$$

$$\frac{\omega_{SS}}{\omega_{REF}} = \frac{1960 \text{ rpm}}{2561.5 \text{ rpm}} = 0.765$$

5. Given the same motor parameters as in question 4, find the system error value in percentage.

The error is equal to:

$$\text{Error} = 85.4\% - \frac{1960 \text{ rpm}}{3000 \text{ rpm}} \times 100\% = 24.8\%$$

6. Compare both calculated error values from question 3 and 5. Which one is lower and why?

The calculated error value is lower in question 5. This is due to the fact that, given the same motor parameters, a higher controller gain results in a steady state closed loop system value ( $\omega_{SS}/\omega_{REF}$ ) closer to 1 and thus, in a steady state speed that is closer to the reference speed.





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