

**Electricity and New Energy**

**Single-Phase AC Power Circuits**

**Course Sample**

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# Safety and Common Symbols

The following safety and common symbols may be used in this course and on the equipment:

Symbol	Description
	<b>DANGER</b> indicates a hazard with a high level of risk which, if not avoided, will result in death or serious injury.
	<b>WARNING</b> indicates a hazard with a medium level of risk which, if not avoided, could result in death or serious injury.
	<b>CAUTION</b> indicates a hazard with a low level of risk which, if not avoided, could result in minor or moderate injury.
	<b>CAUTION</b> used without the <i>Caution, risk of danger</i> sign , indicates a hazard with a potentially hazardous situation which, if not avoided, may result in property damage.
	Caution, risk of electric shock
	Caution, hot surface
	Caution, risk of danger. Consult the relevant user documentation.
	Caution, lifting hazard
	Caution, belt drive entanglement hazard
	Caution, chain drive entanglement hazard
	Caution, gear entanglement hazard
	Caution, hand crushing hazard
	Notice, non-ionizing radiation
	Consult the relevant user documentation.
	Direct current

# Safety and Common Symbols

Symbol	Description
	Alternating current
	Both direct and alternating current
	Three-phase alternating current
	Earth (ground) terminal
	Protective conductor terminal
	Frame or chassis terminal
	Equipotentiality
	On (supply)
	Off (supply)
	Equipment protected throughout by double insulation or reinforced insulation
	In position of a bi-stable push control
	Out position of a bi-stable push control

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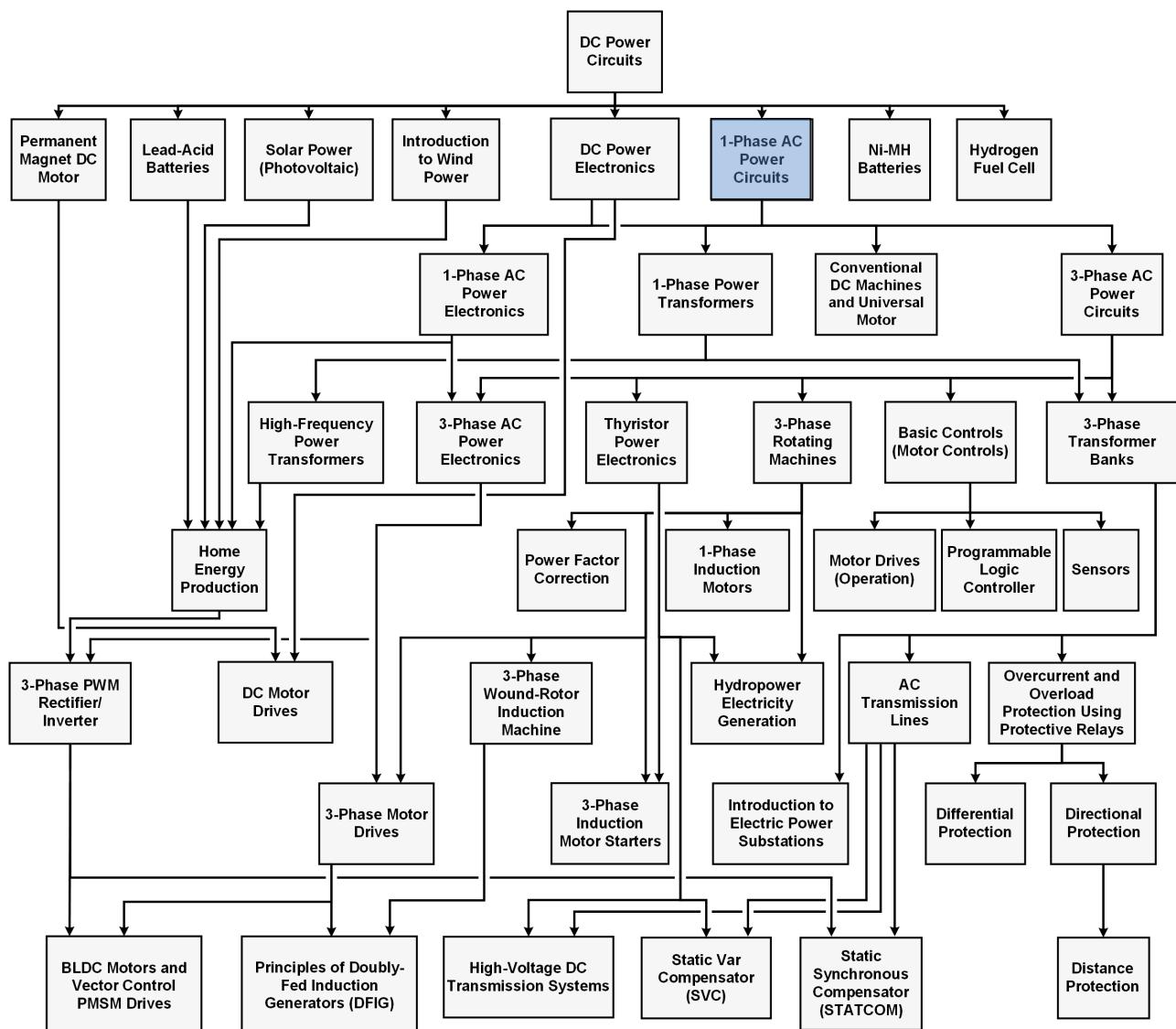
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## Preface

The production of energy using renewable natural resources such as wind, sunlight, rain, tides, geothermal heat, etc., has gained much importance in recent years as it is an effective means of reducing greenhouse gas (GHG) emissions. The need for innovative technologies to make the grid smarter has recently emerged as a major trend, as the increase in electrical power demand observed worldwide makes it harder for the actual grid in many countries to keep up with demand. Furthermore, electric vehicles (from bicycles to cars) are developed and marketed with more and more success in many countries all over the world.

To answer the increasingly diversified needs for training in the wide field of electrical energy, the Electric Power Technology Training Program was developed as a modular study program for technical institutes, colleges, and universities. The program is shown below as a flow chart, with each box in the flow chart representing a course.



## **The Electric Power Technology Training Program.**

# Preface

The program starts with a variety of courses providing in-depth coverage of basic topics related to the field of electrical energy such as ac and dc power circuits, power transformers, rotating machines, ac power transmission lines, and power electronics. The program then builds on the knowledge gained by the student through these basic courses to provide training in more advanced subjects such as motor starters and drives, storage of electrical energy in batteries, home energy production from renewable resources (wind and sunlight), large-scale electricity production from hydropower, protective relaying, and smart-grid technologies (SVC, STATCOM, HVDC transmission systems, etc.).

We invite readers to send us their tips, feedback, and suggestions for improving the course.

Please send these to [did@de.festo.com](mailto:did@de.festo.com).

The authors and Festo Didactic look forward to your comments.

# About This Course

Alternating-current (ac) power systems began to develop quickly in the late 19<sup>th</sup> century, following key developments in the field of electricity, mainly the invention of the polyphase system of electrical distribution by scientist Nikola Tesla, and the development of mathematical analysis of electricity by Charles Steinmetz, James Clerk Maxwell, and William Thomson (Lord Kelvin).

The main advantage of ac power systems is that high amounts of power can be transmitted efficiently over long transmission lines. Step-up transformers are used at the ac power generating point to increase the voltage and decrease the current. The power lost as heat in the resistance of a transmission line increases by the square of the current. Therefore, ac power is transmitted at very high voltages and low currents to reduce power losses in the line resistance to a minimum. At the receiving end of the line, step-down transformers reduce the voltage and increase the current to levels compatible with residential or industrial equipment.

Today ac power systems are used throughout the world for driving motors and powering electric equipment in transport, heating, lighting, communications, and computation.

This course, *Single-Phase AC Power Circuit*, introduces students to the fundamentals of alternating current, such as the sine wave, period and frequency, phase angle and phase shift, instantaneous and average power, etc. Students then become familiar with the inductor and capacitor. The course continues with more advanced topics such as the impedance, active power, reactive power, apparent power, and power triangle. The course concludes by teaching students how to solve ac power circuits using the impedance calculation method or the power triangle method.



**Most lighting in urban centers is powered using single-phase alternative current.**

# About This Course

## Safety considerations

Safety symbols that may be used in this course and on the equipment are listed in the Safety and Common Symbols table at the beginning of this document.

Safety procedures related to the tasks that you will be asked to perform are indicated in each exercise.

Make sure that you are wearing appropriate protective equipment when performing the tasks. You should never perform a task if you have any reason to think that a manipulation could be dangerous for you or your teammates.

## Prerequisite

As a prerequisite to this course, you should have completed course *DC Power Circuits*.

## Systems of units

Units are expressed using the International System of Units (SI) followed by units expressed in the U.S. customary system of units (between parentheses).

# To the Instructor

You will find in this Instructor Guide all the elements included in the Student Manual together with the answers to all questions, results of measurements, graphs, explanations, suggestions, and, in some cases, instructions to help you guide the students through their learning process. All the information that applies to you is placed between markers and appears in red.

## **Accuracy of measurements**

The numerical results of the hands-on exercises may differ from one student to another. For this reason, the results and answers given in this course should be considered as a guide. Students who correctly perform the exercises should expect to demonstrate the principles involved and make observations and measurements similar to those given as answers.

## **Equipment installation**

In order for students to be able to perform the exercises in the Student Manual, the Electric Power Technology Training Equipment must have been properly installed, according to the instructions given in the user guide Electric Power Technology Training Equipment.



Sample  
Extracted from  
Instructor Guide



## Phase Angle and Phase Shift

### EXERCISE OBJECTIVE

When you have completed this exercise, you will know what a phase angle is and how the phase angle modifies the initial displacement of a sine wave. You will be able to determine the phase shift between two sine waves, either by comparing their phase angles or by determining their separation in time. You will also know how to distinguish a leading phase shift from a lagging phase shift.

### DISCUSSION OUTLINE

The Discussion of this exercise covers the following points:

- Phase angle
- Phase shift

### DISCUSSION

#### Phase angle

As you have seen in Exercise 1-1, the graphical representation of a sine wave can be expressed by the following equation:

$$a(t) = A \sin(\omega t) \quad (1-8)$$

where  $a(t)$  is the instantaneous value of the sine wave at a given instant  $t$ .

$A$  is the amplitude of the sine wave.

$\omega$  is the angular velocity, expressed in radians per second (rad/s).

$t$  is the time, expressed in seconds (s).

This equation assumes that the sine wave cycle begins at the exact moment when  $t = 0$  (as is shown in Figure 1-10). As you will see later, this is not always the case. To represent the initial position of the sine wave, the notion of **phase angle**  $\theta$  is introduced in the equation below:

$$a(t) = A \sin(\omega t + \theta) \quad (1-9)$$

where  $\theta$  is the phase angle of the sine wave, expressed in degrees ( $^{\circ}$ ) or radians (rad).

From Equation (1-9), it is easy to observe that the initial value (i.e., the value at  $t = 0$ ) of the sine wave depends entirely on the phase angle  $\theta$  because the term  $\omega t$  equals 0 at  $t = 0$ . In other words, the phase angle  $\theta$  determines by how much the value of a sine wave differs from 0 at time  $t = 0$ , and thus, the position in time of the sine wave.

Figure 1-10 shows a sine wave with a phase angle  $\theta$  of  $0^\circ$ . The initial value of this sine wave is 0 because  $A \sin(\omega \cdot 0 + 0) = 0$ . This sine wave is identical to those seen in Exercise 1-1, as a phase angle value of  $0^\circ$  was implied by the absence of  $\theta$  in the equations given in Exercise 1-1.

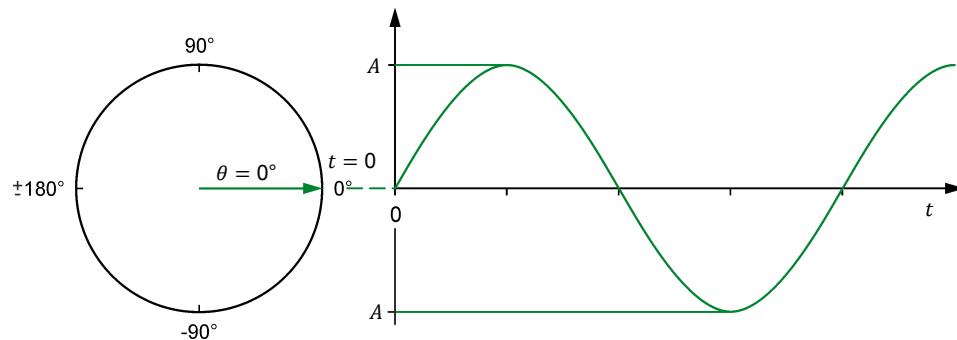


Figure 1-10. Sine wave with a phase angle  $\theta$  of  $0^\circ$ .

Figure 1-11 shows a sine wave with a phase angle  $\theta$  of  $45^\circ$ . As you can see from the figure, a positive phase angle ( $0^\circ$  to  $180^\circ$ ) results in the sine wave having a positive instantaneous value when  $t = 0$ . In other words, a positive phase angle shifts the sine wave toward the left, i.e., advances the sine wave in time.

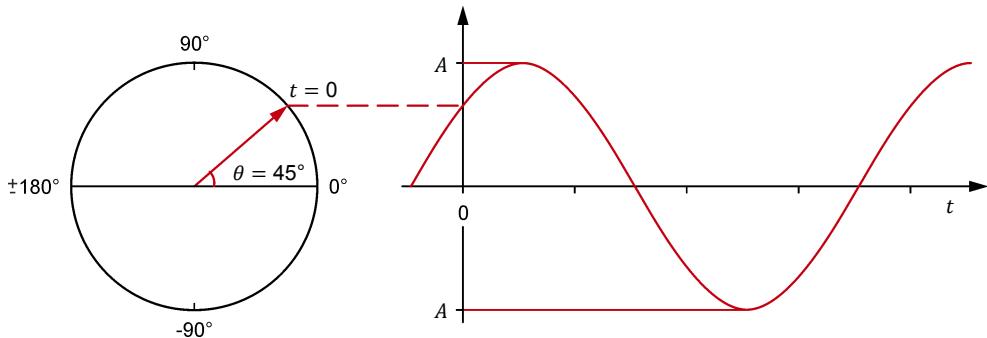


Figure 1-11. Sine wave with a phase angle  $\theta$  of  $45^\circ$ .

Figure 1-12 shows a sine wave with a phase angle  $\theta$  of  $-60^\circ$ . A negative phase angle ( $0^\circ$  to  $-180^\circ$ ) results in the sine wave having a negative instantaneous value when  $t = 0$ . In other words, a negative phase angle shifts the sine wave toward the right, i.e., delays the sine wave in time.

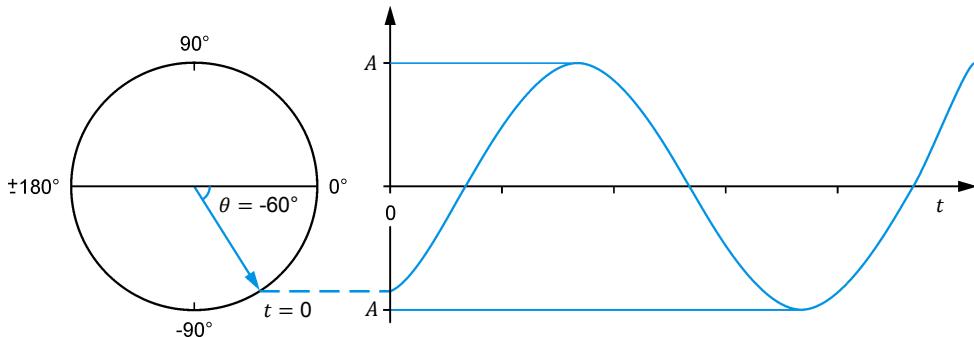


Figure 1-12. Sine wave with a phase angle  $\theta$  of  $-60^\circ$ .

Figure 1-10 to Figure 1-12 also show the phasor representations of the sine waves at time  $t = 0$ . Notice that, in each figure, the vertical distance between the tip of the rotating phasor representing the sine wave matches the instantaneous value of the sine wave at  $t = 0$ .

### Phase shift

When comparing two sine waves having the same frequency, the difference between their respective phase angles is called the **phase shift** and is expressed in degrees ( $^{\circ}$ ) or radians (rad). The magnitude of the phase shift indicates the extent of separation in time between the two sine waves, while the polarity of the phase shift (positive or negative) indicates the relationship in time between the two sine waves (**leading** or **lagging**). The sine wave amplitude value has no effect on the phase shift, as it does not change the period nor the frequency of the sine wave. Sine waves with different frequencies and, as an extension, different periods, cannot be compared by using their phase angles as their cycles do not correspond.

The phase shift between two sine waves is expressed as an angle representing a portion of a complete cycle of the sine waves. One of the two sine waves is used as the reference for phase shift measurements. The phase shift is calculated by subtracting the phase angle  $\theta_{Ref.}$  of the reference sine wave from the phase angle  $\theta$  of the sine wave of interest. This is written as an equation below.

$$\text{Phase shift} = \theta - \theta_{Ref.} \quad (1-10)$$

where  $\theta$  is the phase angle of the sine wave of interest, expressed in degrees ( $^{\circ}$ ) or radians (rad).

$\theta_{Ref.}$  is the phase angle of the reference sine wave, expressed in degrees ( $^{\circ}$ ) or radians (rad).

Figure 1-13 is an example showing how the phase shift between two sine waves (X and Y) can be calculated using their phase angles.

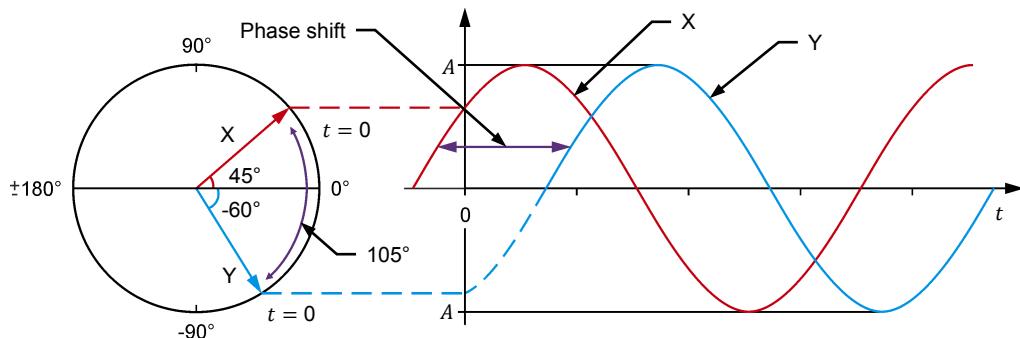


Figure 1-13. Phase shift between two sine waves with phase angles of  $45^{\circ}$  and  $-60^{\circ}$ .

In the figure, sine wave X has a phase angle  $\theta$  of  $45^{\circ}$ , while sine wave Y has a phase angle  $\theta$  of  $-60^{\circ}$ . Depending on which sine wave is used as the reference, the phase shift can be  $+105^{\circ}$  or  $-105^{\circ}$ . When sine wave X is considered as the reference, the phase shift of sine wave Y with respect to sine wave X is  $-105^{\circ}$  ( $-60^{\circ} - 45^{\circ} = -105^{\circ}$ ). The minus sign in this phase shift value indicates

that sine wave Y lags reference sine wave X. For this reason, this phase shift value can also be expressed as 105° lagging. Conversely, when sine wave Y is considered as the reference, the phase shift of sine wave X with respect to sine wave Y is +105° ( $45^\circ - (-60)^\circ = +105^\circ$ ). The plus sign in this phase shift value indicates that sine wave X leads reference sine wave Y. For this reason, this phase shift value can also be expressed as 105° leading. Note that whenever two sine waves have different phase angles, the phase shift value is not zero, and thus, these sine waves are said to be out of phase.

It is possible to determine the phase shift between two sine waves of the same frequency without knowing their respective phase angles  $\theta$ . The following equation is used:

$$\text{Phase shift} = \frac{d}{T} \times 360^\circ = \frac{d}{T} \times 2\pi \text{ rad} \quad (1-11)$$

where  $d$  is the time interval between a given reference point on each of the two sine waves, expressed in seconds (s).

$T$  is the period of the sine waves, expressed in seconds (s).

This equation shows in a concrete way why it is not possible to calculate the phase shift between two sine waves having different frequencies  $f$ , as a common period  $T$  ( $T = 1/f$ ) is needed for the equation to be valid.

Consider, for example, the sine waves shown in Figure 1-14. Using Equation (1-11), the phase shift between the two sine waves is equal to:

$$\text{Phase shift} = \frac{d}{T} \times 360^\circ = \frac{3.33 \text{ ms}}{20.0 \text{ ms}} \times 360^\circ = 60^\circ$$

When sine wave 1 is used as the reference, the phase shift is lagging because sine wave 2 is delayed with respect to sine wave 1. Conversely, when sine wave 2 is considered as the reference, the phase shift is leading because sine wave 1 is in advance with respect to sine wave 2.

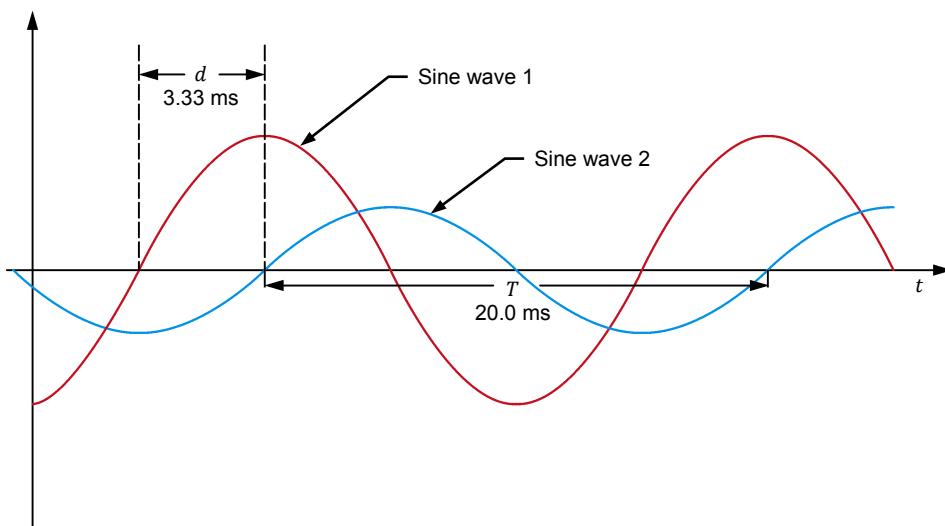


Figure 1-14. Phase shift between two sine waves having the same frequency.

## PROCEDURE OUTLINE

The Procedure is divided into the following sections:

- Setup and connections
- Measuring the phase shift between two voltage sine waves in a resistor-inductor (RL) circuit
- Measuring the phase shift between two voltage sine waves in a resistor-capacitor (RC) circuit

## PROCEDURE

### **WARNING**



High voltages are present in this laboratory exercise. Do not make or modify any banana jack connections with the power on unless otherwise specified.

### Setup and connections

*In this section, you will connect an ac circuit containing an inductor and a resistor in series and set up the equipment to measure the source voltage  $E_S$ , as well as the voltage across the resistor  $E_R$ .*

1. Refer to the Equipment Utilization Chart in Appendix A to obtain the list of equipment required to perform this exercise.

Install the required equipment in the [Workstation](#).

2. Make sure that the main power switch on the [Four-Quadrant Dynamometer/Power Supply](#) is set to the [O](#) (off) position, then connect its [Power Input](#) to an ac power outlet.

3. Connect the [Power Input](#) of the [Data Acquisition and Control Interface](#) to a 24 V ac power supply. Turn the 24 V ac power supply on.

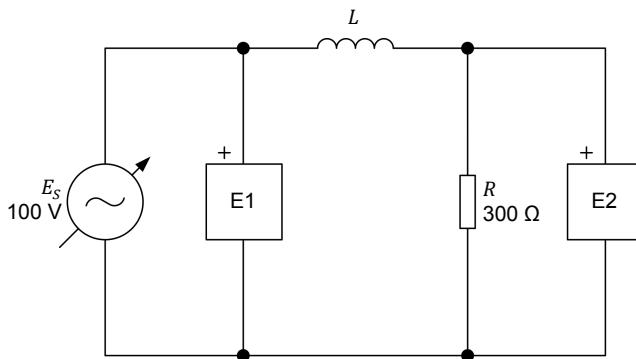
4. Turn the [Four-Quadrant Dynamometer/Power Supply](#) on, then set the [Operating Mode](#) switch to [Power Supply](#). This setting allows the [Four-Quadrant Dynamometer/Power Supply](#) to operate as a power supply.

5. Connect the USB port of the [Data Acquisition and Control Interface](#) to a USB port of the host computer.

Connect the USB port of the [Four-Quadrant Dynamometer/Power Supply](#) to a USB port of the host computer.

6. Turn the host computer on, then start the [LVDAC-EMS](#) software.

7. In the LVDAC-EMS Start-Up window, make sure that the Data Acquisition and Control Interface and the Four-Quadrant Dynamometer/Power Supply are detected. Make sure that the Computer-Based Instrumentation function for the Data Acquisition and Control Interface is available. Select the network voltage and frequency that correspond to the voltage and frequency of your local ac power network, then click the OK button to close the LVDAC-EMS Start-Up window.
  
8. Set up the circuit shown in Figure 1-15. This circuit contains a resistor  $R$  and an inductor  $L$ . Inductors are studied in the next unit of this manual.



**Figure 1-15. AC circuit with a resistor and an inductor.**

The value of inductor  $L$  in the circuit of Figure 1-15 is referred to as the inductance and is expressed in henries (H). The inductance value to be used depends on the frequency of the ac power source as is indicated in Table 1-2.



As indicated in Appendix A, use the Inductive Load module to obtain the required inductance when the ac power network frequency is 60 Hz. Use the Inductive and Capacitive Loads module to obtain the required inductance when the ac power network frequency is 50 Hz.

**Table 1-2. Inductance values for 50 and 60 Hz frequencies.**

Power source frequency (Hz)	Inductance (H)
50	0.96
60	0.80

Make the necessary switch settings on the Resistive Load in order to obtain the resistance values required.



Appendix C of this manual lists the switch settings to implement on the Resistive Load in order to obtain various resistance values.

Make the necessary connections and switch settings on the Inductive Load (or on the Inductive and Capacitive Loads) in order to obtain the inductance value required.



If necessary, ask your instructor to assist you to obtain the inductance value required.

Use inputs *E1* and *E2* of the Data Acquisition and Control Interface to measure the source voltage  $E_S$  and the voltage across the resistor  $E_R$ , respectively.

9. In LVDAC-EMS, open the Four-Quadrant Dynamometer/Power Supply window, then make the following settings:

- Set the *Function* parameter to *AC Power Source*.
- Make sure that the *Voltage Control* parameter is set to *Knob*. This allows the ac power source to be controlled manually.
- Set the *Voltage (V at no load)* parameter to 100 V.
- Set the *Frequency* parameter to the frequency of your local ac power network.
- Leave the other parameters set as they are.

### **Measuring the phase shift between two voltage sine waves in a resistor-inductor (RL) circuit**

*In this section, you will observe the waveforms (sine waves) of the source voltage  $E_S$  and the resistor voltage  $E_R$ , using the Oscilloscope to determine the phase shift between the two sine wave voltages. Then, using the Phasor Analyzer, you will measure the phase shift between the source voltage phasor and the resistor voltage phasor and compare it to the phase shift determined from the voltage waveforms.*



*As you will see later, due to the presence of an inductor in the circuit, the circuit current lags behind the source voltage. As a result, the voltage  $E_R$  measured across the resistor is out of phase with respect to the source voltage  $E_S$ .*

10. In LVDAC-EMS, open the Metering window. Set meters *E1* and *E2* to measure the rms values of the source voltage  $E_S$  and voltage  $E_R$  across the resistor  $R$ , respectively.

In the Four-Quadrant Dynamometer/Power Supply window, enable the ac power source. Readjust the value of the *Voltage (V at no load)* parameter so that the ac power source voltage  $E_S$  (indicated by meter *E1* in the Metering window) is equal to 100 V.

11. In LVDAC-EMS, open the Oscilloscope and display  $E_S$  (input *E1*) and  $E_R$  (input *E2*) on channels 1 and 2, respectively. If necessary, set the time base so as to display at least two cycles of the sine waves. Place the traces of the two channels at the same vertical position.

- 12.** Measure the period  $T$  of the source voltage  $E_S$  using the **Oscilloscope** then record the value below.

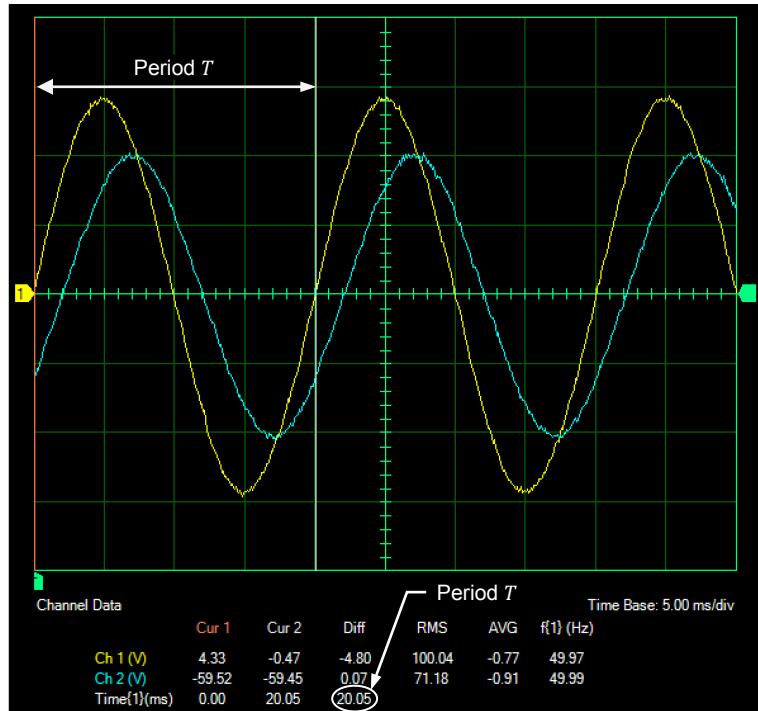


To obtain an accurate measurement, you can use the vertical cursors of the **Oscilloscope** to measure the period or any other time interval.

Period  $T$  = \_\_\_\_\_ ms

50 Hz: Period  $T = 20.05$  ms. The results are shown in the following figure.

Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising

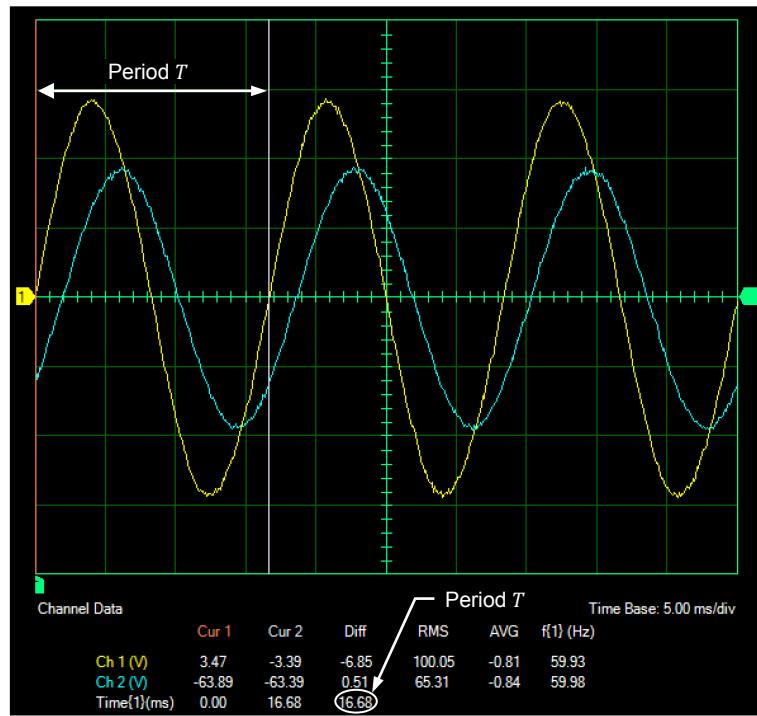


Period  $T$  of the waveform of the source voltage  $E_S$  at a frequency of 50 Hz.

60 Hz: Period  $T = 16.68$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input .....	E1
Channel-1 Scale .....	.50 V/div
Channel-1 Coupling .....	DC
Channel-2 Input .....	E2
Channel-2 Scale .....	.50 V/div
Channel-2 Coupling .....	DC
Display Filtering .....	On
Show Cursors .....	Vertical
Trigger Type .....	Software
Time Base.....	5 ms/div
Trigger Source .....	Ch1
Trigger Level.....	0
Trigger Slope .....	Rising



Period  $T$  of the waveform of the source voltage  $E_s$  at a frequency of 60 Hz.

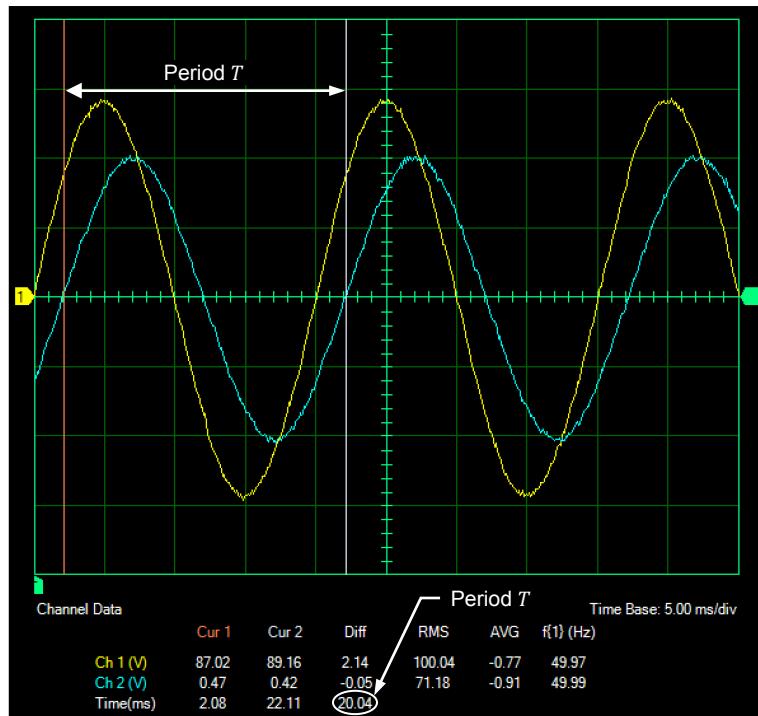
- 13.** Measure the period  $T$  of the resistor voltage  $E_R$  using the Oscilloscope then record the value below.

Period  $T$  = \_\_\_\_\_ ms

50 Hz: Period  $T = 20.04$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input	.....	E1
Channel-1 Scale	.....	50 V/div
Channel-1 Coupling	.....	DC
Channel-2 Input	.....	E2
Channel-2 Scale	.....	50 V/div
Channel-2 Coupling	.....	DC
Display Filtering	.....	On
Show Cursors	.....	Vertical
Trigger Type	.....	Software
Time Base	.....	5 ms/div
Trigger Source	.....	Ch1
Trigger Level	.....	0
Trigger Slope	.....	Rising

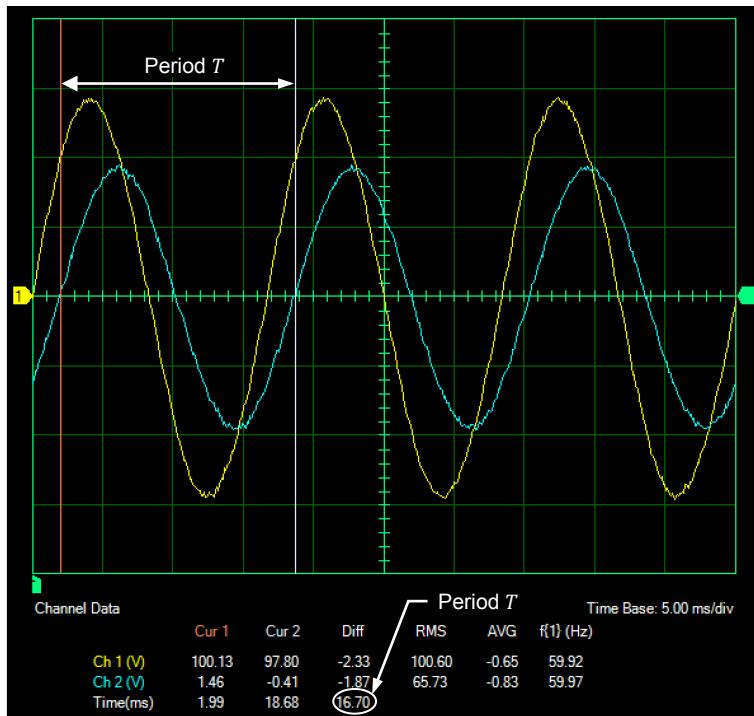


Period  $T$  of the waveform of the resistor voltage  $E_R$  at a frequency of 50 Hz.

60 Hz: Period  $T = 16.70$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input .....	E1
Channel-1 Scale .....	.50 V/div
Channel-1 Coupling .....	DC
Channel-2 Input .....	E2
Channel-2 Scale .....	.50 V/div
Channel-2 Coupling .....	DC
Display Filtering .....	On
Show Cursors .....	Vertical
Trigger Type .....	Software
Time Base.....	5 ms/div
Trigger Source .....	Ch1
Trigger Level.....	0
Trigger Slope .....	Rising



Period  $T$  of the waveform of the resistor voltage  $E_R$  at a frequency of 60 Hz.

14. Compare the period  $T$  of the resistor voltage  $E_R$  measured in the previous step with the period  $T$  of the source voltage  $E_S$  recorded in step 12. Are the values close to each other?

Yes     No

Yes

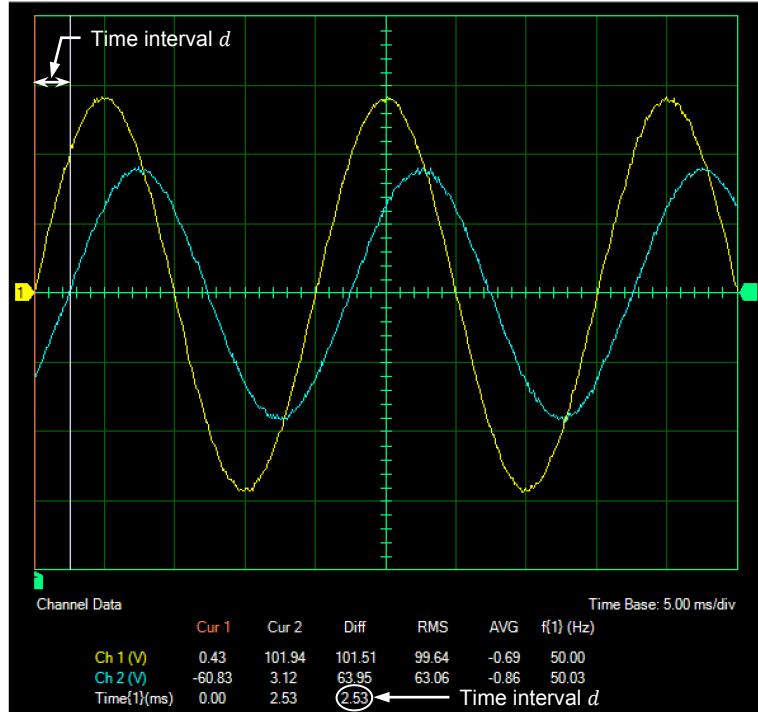
- 15.** Measure the time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$  by using the Oscilloscope.

Time interval  $d = \underline{\hspace{2cm}}$  ms

50 Hz: Time interval  $d = 2.53$  ms. The results are shown in the following figure.

#### Oscilloscope Settings

Channel-1 Input	E1
Channel-1 Scale	.50 V/div
Channel-1 Coupling	DC
Channel-2 Input	E2
Channel-2 Scale	.50 V/div
Channel-2 Coupling	DC
Display Filtering	On
Show Cursors	Vertical
Trigger Type	Software
Time Base	5 ms/div
Trigger Source	Ch1
Trigger Level	0
Trigger Slope	Rising

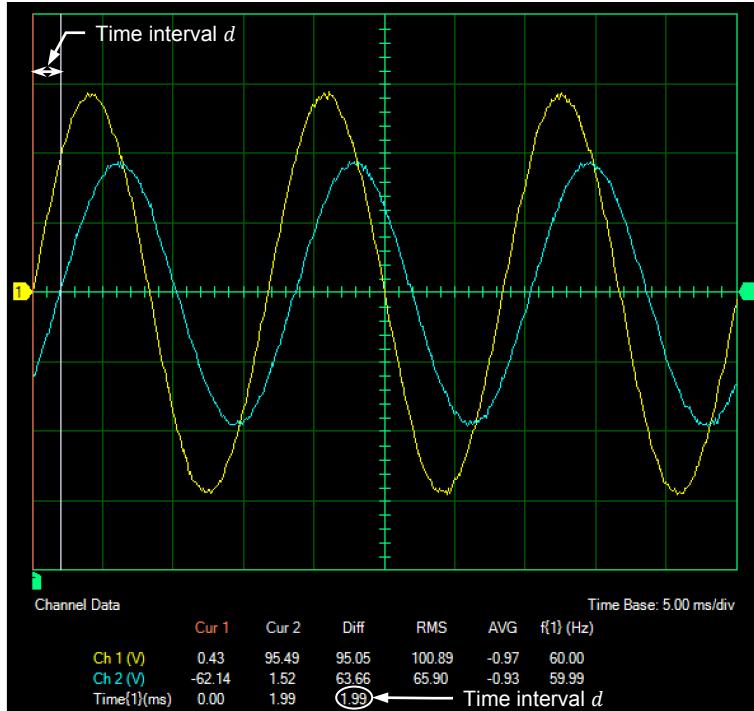


Time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$  at a frequency of 50 Hz.

60 Hz: Time interval  $d = 1.99$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input .....	E1
Channel-1 Scale .....	.50 V/div
Channel-1 Coupling .....	DC
Channel-2 Input .....	E2
Channel-2 Scale .....	.50 V/div
Channel-2 Coupling .....	DC
Display Filtering .....	On
Show Cursors .....	Vertical
Trigger Type .....	Software
Time Base.....	5 ms/div
Trigger Source .....	Ch1
Trigger Level .....	0
Trigger Slope .....	Rising



Time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$  at a frequency of 60 Hz.

16. Using Equation (1-11), calculate the phase shift between the source voltage  $E_S$  and the resistor voltage  $E_R$ . Consider the source voltage waveform as the reference.

$$\text{Phase shift} = \underline{\hspace{2cm}}^\circ$$

50 Hz:  $\text{Phase shift} = \frac{d}{T} \cdot 360^\circ = \frac{2.53 \text{ ms}}{20.1 \text{ ms}} \cdot 360^\circ = 45.3^\circ$

$\text{Phase shift} = 45.3^\circ$  lagging or  $-45.3^\circ$

60 Hz:  $\text{Phase shift} = \frac{d}{T} \cdot 360^\circ = \frac{1.99 \text{ ms}}{16.7 \text{ ms}} \cdot 360^\circ = 42.9^\circ$

$\text{Phase shift} = 42.9^\circ$  lagging or  $-42.9^\circ$

17. Is the resistor voltage  $E_R$  leading or lagging the source voltage  $E_S$ ?

The resistor voltage  $E_R$  is lagging the source voltage  $E_S$ .

- 18.** In LVDAC-EMS, open the Phasor Analyzer and display the source voltage  $E_S$  (input **E1**) and resistor voltage  $E_R$  (input **E2**). Set the Reference Phasor parameter to **E1**. Measure the phase angles  $\theta_{ES}$  and  $\theta_{ER}$  of the voltage phasors.

Phase angle  $\theta_{ES} = \underline{\hspace{2cm}}$  °

Phase angle  $\theta_{ER} = \underline{\hspace{2cm}}$  °

From these values, calculate the phase shift between the phasors of the source voltage  $E_S$  and resistor voltage  $E_R$ , using the source voltage phasor as the reference.

Phase shift =  $\underline{\hspace{2cm}}$  °

50 Hz: Phase angle  $\theta_{ES} = 0^\circ$

Phase angle  $\theta_{ER} = -43.8^\circ$

Phase shift =  $43.8^\circ$  lagging or  $-43.8^\circ$

60 Hz: Phase angle  $\theta_{ES} = 0^\circ$

Phase angle  $\theta_{ER} = -42.3^\circ$

Phase shift =  $42.3^\circ$  lagging or  $-42.3^\circ$

- 19.** Compare the phase shift you determined from the voltage sine waves to the phase shift you measured from the corresponding voltage phasors. Are both values close to each other?

Yes     No

Yes

### Measuring the phase shift between two voltage sine waves in a resistor-capacitor (RC) circuit

In this section, you will replace the inductor used in the previous section by a capacitor. Using the Oscilloscope, you will determine the phase shift between the two voltage sine waves. Then, using the Phasor Analyzer, you will measure the phase shift between the source voltage phasor and the resistor voltage phasor and compare it to the phase shift you determined from the voltage waveforms.



As you will see later, due to the presence of a capacitor in the circuit, the circuit current leads the source voltage. As a result, the resistor voltage  $E_R$  is out of phase with respect to the source voltage  $E_S$ .

- 20.** In the Four-Quadrant Dynamometer/Power Supply window, disable the ac power source.

- 21.** Modify the circuit so that it is as shown in Figure 1-16 (replace the inductor by a capacitor). This circuit contains a resistor  $R$  and a capacitor  $C$ . Capacitors are studied in the next unit of this manual.

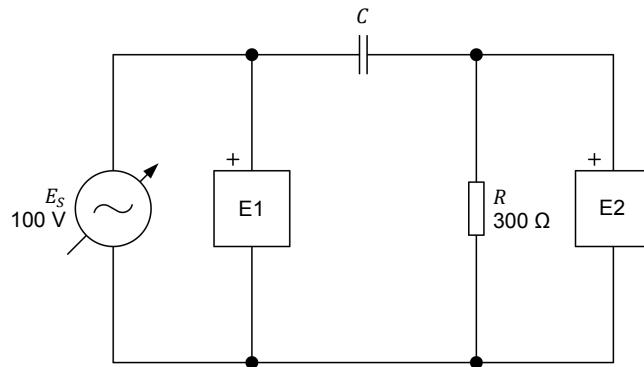


Figure 1-16. AC circuit with a resistor and a capacitor.

The value of capacitor  $C$  in the circuit of Figure 1-16 is referred to as the capacitance and is expressed in microfarads ( $\mu\text{F}$ ). The capacitance value to be used depends on the frequency of the ac power source as is indicated in Table 1-3.

As indicated in Appendix A, use the [Capacitive Load](#) module to obtain the required capacitance when the ac power network frequency is 60 Hz. Use the [Inductive and Capacitive Loads](#) module to obtain the required capacitance when the ac power network frequency is 50 Hz.

Table 1-3. Capacitance values for 50 and 60 Hz frequencies.

Power source frequency (Hz)	Capacitance ( $\mu\text{F}$ )
50	5.3
60	4.4

Make the necessary switch settings on the [Resistive Load](#) in order to obtain the resistance values required.

Appendix C of this manual lists the switch settings to implement on the [Resistive Load](#) in order to obtain various resistance values.

Make the necessary connections and switch settings on the [Capacitive Load](#) (or on the [Inductive and Capacitive Loads](#)) in order to obtain the capacitance value required.

If necessary, ask your instructor to assist you to obtain the capacitance value required.

- 22.** In the [Four-Quadrant Dynamometer/Power Supply](#) window, enable the ac power source. Readjust the value of the [Voltage \(V at no load\)](#) parameter, if necessary, so that the ac power source voltage  $E_s$  (indicated by meter [E1](#) in the [Metering](#) window) is equal to 100 V.

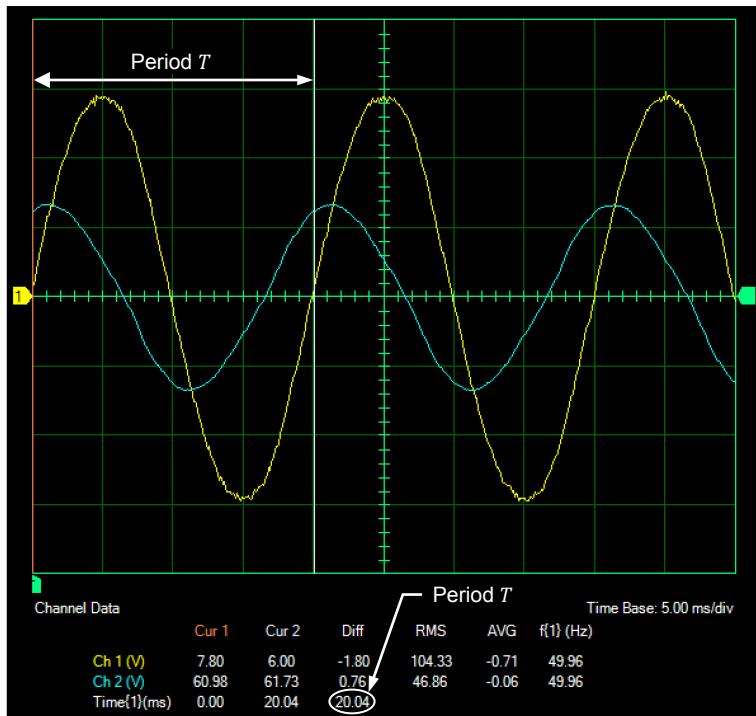
- 23.** Measure the period  $T$  of the source voltage using the Oscilloscope then record the value below.

Period  $T$  = \_\_\_\_\_ ms

50 Hz: Period  $T = 20.04$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input	.....	E1
Channel-1 Scale	.....	50 V/div
Channel-1 Coupling	.....	DC
Channel-2 Input	.....	E2
Channel-2 Scale	.....	50 V/div
Channel-2 Coupling	.....	DC
Display Filtering	.....	On
Show Cursors	.....	Vertical
Trigger Type	.....	Software
Time Base	.....	5 ms/div
Trigger Source	.....	Ch1
Trigger Level	.....	0
Trigger Slope	.....	Rising

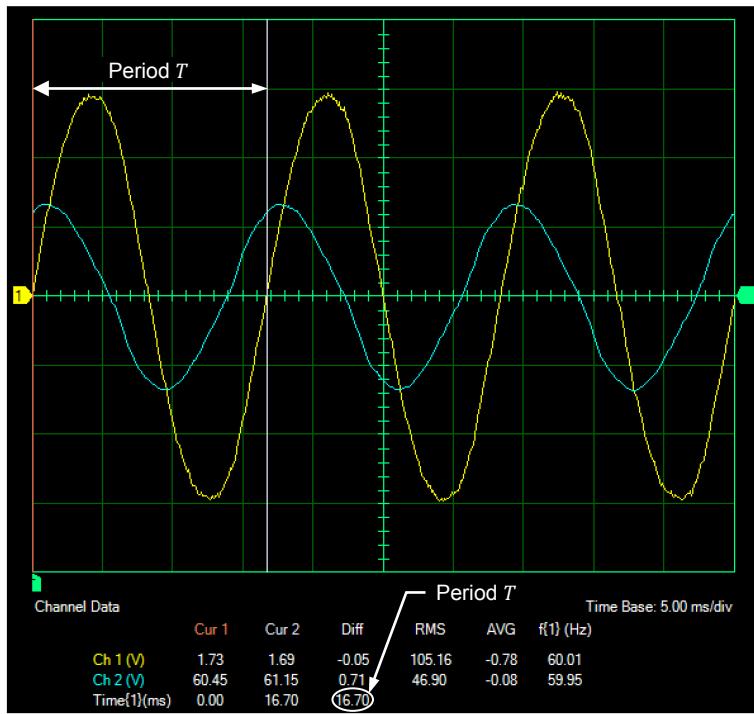


Period  $T$  of the waveform of the source voltage  $E_S$  at a frequency of 50 Hz.

60 Hz: Period  $T = 16.70$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input .....	E1
Channel-1 Scale .....	.50 V/div
Channel-1 Coupling .....	DC
Channel-2 Input .....	E2
Channel-2 Scale .....	.50 V/div
Channel-2 Coupling .....	DC
Display Filtering .....	On
Show Cursors .....	Vertical
Trigger Type .....	Software
Time Base.....	5 ms/div
Trigger Source .....	Ch1
Trigger Level.....	0
Trigger Slope .....	Rising



Period  $T$  of the waveform of the source voltage  $E_s$  at a frequency of 60 Hz.

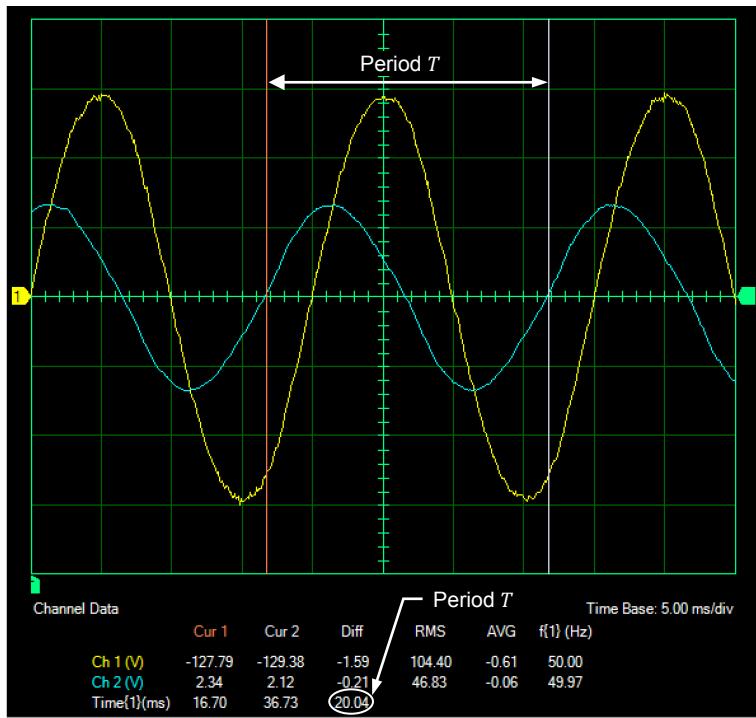
- 24.** Measure the period  $T$  of the resistor voltage  $E_R$  using the Oscilloscope then record the value below.

Period  $T$  = \_\_\_\_\_ ms

50 Hz: Period  $T = 20.04$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input	.....	E1
Channel-1 Scale	.....	50 V/div
Channel-1 Coupling	.....	DC
Channel-2 Input	.....	E2
Channel-2 Scale	.....	50 V/div
Channel-2 Coupling	.....	DC
Display Filtering	.....	On
Show Cursors	.....	Vertical
Trigger Type	.....	Software
Time Base	.....	5 ms/div
Trigger Source	.....	Ch1
Trigger Level	.....	0
Trigger Slope	.....	Rising

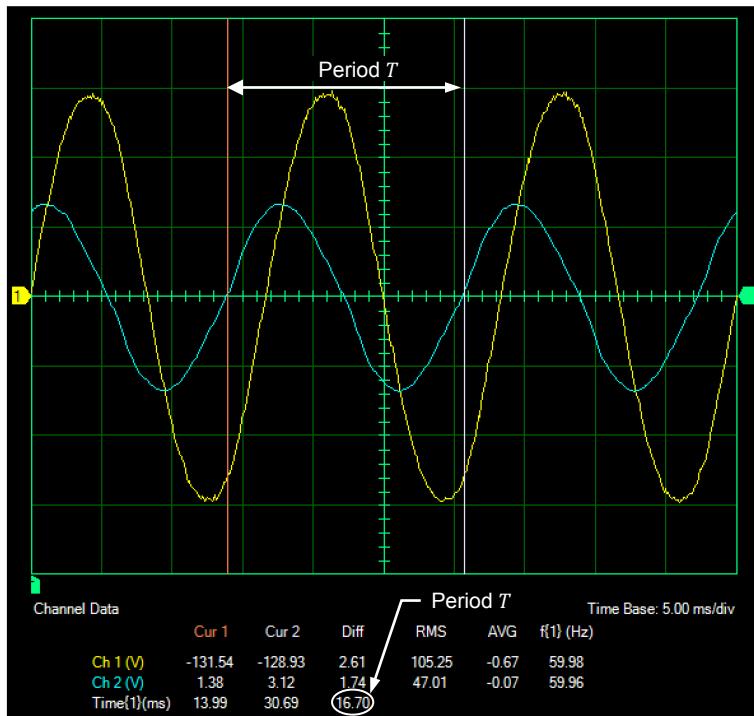


Period  $T$  of the waveform of the resistor voltage  $E_S$  at a frequency of 50 Hz.

60 Hz: Period  $T = 16.70$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input	.....	E1
Channel-1 Scale	.....	.50 V/div
Channel-1 Coupling	.....	DC
Channel-2 Input	.....	E2
Channel-2 Scale	.....	.50 V/div
Channel-2 Coupling	.....	DC
Display Filtering	.....	On
Show Cursors	.....	Vertical
Trigger Type	.....	Software
Time Base	.....	5 ms/div
Trigger Source	.....	Ch1
Trigger Level	.....	0
Trigger Slope	.....	Rising



Period  $T$  of the waveform of the resistor voltage  $E_R$  at a frequency of 60 Hz.

25. Compare the period  $T$  of the resistor voltage  $E_R$  measured in the previous step with the period  $T$  of the source voltage  $E_S$  recorded in step 23. Are the values close to each other?

Yes     No

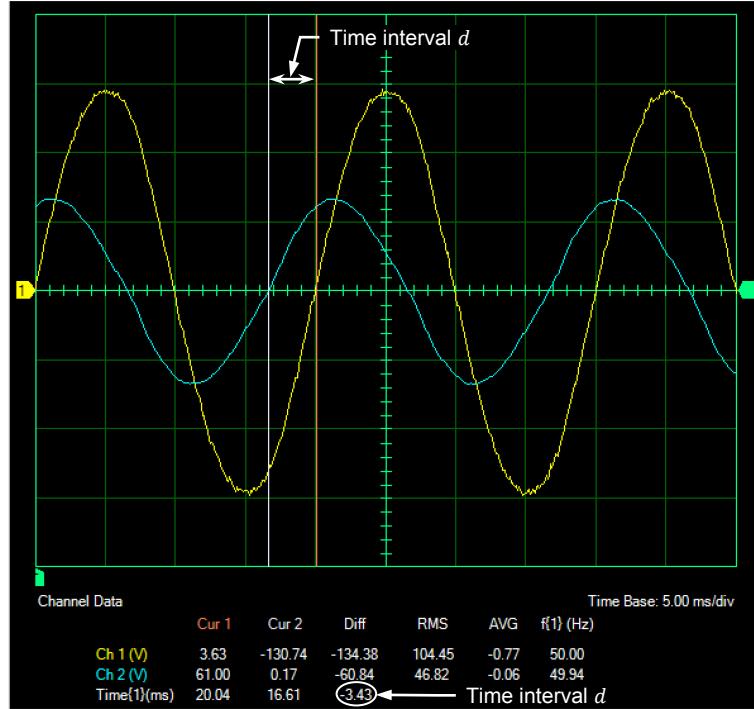
Yes

- 26.** Measure the time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$ .

Time interval  $d = \underline{\hspace{2cm}}$  ms

50 Hz: Time interval  $d = 3.43$  ms. The results are shown in the following figure.

Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising

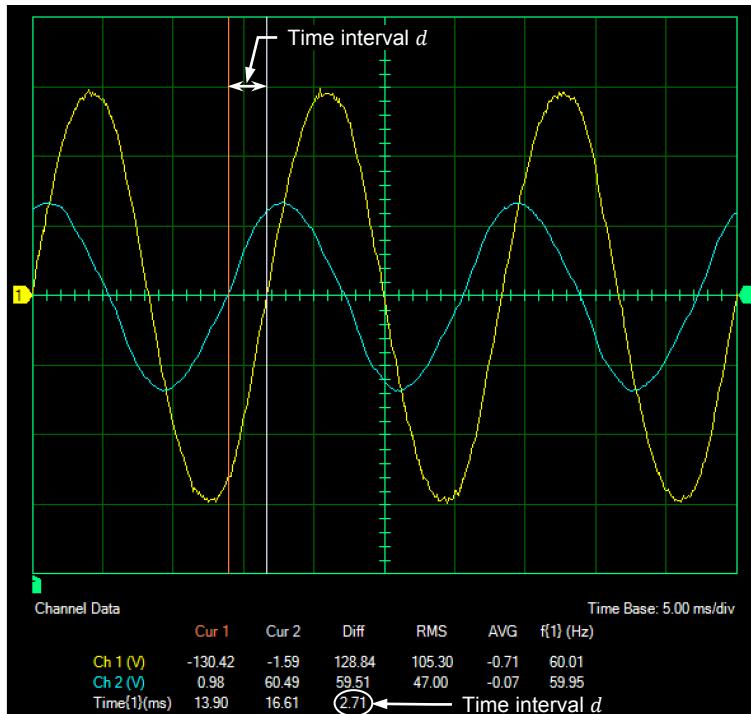


Time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$  at a frequency of 50 Hz.

60 Hz: Time interval  $d = 2.71$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input .....	E1
Channel-1 Scale .....	.50 V/div
Channel-1 Coupling .....	DC
Channel-2 Input .....	E2
Channel-2 Scale .....	.50 V/div
Channel-2 Coupling .....	DC
Display Filtering .....	On
Show Cursors .....	Vertical
Trigger Type .....	Software
Time Base.....	5 ms/div
Trigger Source .....	Ch1
Trigger Level .....	0
Trigger Slope .....	Rising



Time interval  $d$  between the waveforms of the source voltage  $E_s$  and resistor voltage  $E_R$  at a frequency of 60 Hz.

27. Using Equation (1-11), calculate the phase shift between the source voltage  $E_s$  and the resistor voltage  $E_R$ . Consider the source voltage waveform as the reference.

$$\text{Phase shift} = \underline{\hspace{2cm}}^\circ$$

50 Hz:  $\text{Phase shift} = \frac{d}{T} \cdot 360^\circ = \frac{3.43 \text{ ms}}{20.0 \text{ ms}} \cdot 360^\circ = 61.7^\circ$

$\text{Phase shift} = 61.7^\circ$  leading or  $61.7^\circ$

60 Hz:  $\text{Phase shift} = \frac{d}{T} \cdot 360^\circ = \frac{2.71 \text{ ms}}{16.7 \text{ ms}} \cdot 360^\circ = 58.4^\circ$

$\text{Phase shift} = 58.4^\circ$  leading or  $58.4^\circ$

28. Is the resistor voltage  $E_R$  leading or lagging the source voltage  $E_s$ ?

The resistor voltage  $E_R$  is leading the source voltage  $E_s$ .

- 29.** In the Phasor Analyzer, measure the phase angles  $\theta_{ES}$  and  $\theta_{ER}$  of the voltage phasors.

Phase angle  $\theta_{ES} = \underline{\hspace{2cm}}$  °

Phase angle  $\theta_{ER} = \underline{\hspace{2cm}}$  °

From these values, calculate the phase shift between the phasors of the source voltage  $E_S$  and resistor voltage  $E_R$ , using the source voltage phasor as the reference.

Phase shift =        °

50 Hz:      Phase angle  $\theta_{ES} = 0^\circ$

Phase angle  $\theta_{ER} = 63.4^\circ$

Phase shift =  $63.4^\circ$

60 Hz:      Phase angle  $\theta_{ES} = 0^\circ$

Phase angle  $\theta_{ER} = 63.5^\circ$

Phase shift =  $63.5^\circ$

- 30.** Compare the phase shift you determined from the voltage sine waves to the phase shift you measured from the corresponding voltage phasors. Are both values close to each other?

Yes       No

Yes

- 31.** In the Four-Quadrant Dynamometer/Power Supply window, disable the ac power source.

- 32.** Close LVDAC-EMS, then turn off all the equipment. Disconnect all leads and return them to their storage location.

## CONCLUSION

In this exercise, you saw how the phase angle modifies the value of a sine wave at time  $t = 0$ , and thus, the position in time of the sine wave. You observed the effects of positive and negative phase angles on the relative position in time of a sine wave. You were introduced to the notion of phase shift. You learned how to calculate and measure the phase shift between two sine waves and to differentiate between a lagging and a leading phase shift.

**REVIEW QUESTIONS**

1. What is the effect of the phase angle on the graphical representation of a sine wave?

The phase angle determines the value of a sine wave when  $t = 0$  s, and thus, the position in time of the sine wave.

2. A sine wave has a phase angle  $\theta$  of  $72^\circ$ . Will this sine wave reach its maximum value before, after or at the same time as a second waveform having a phase angle  $\theta$  of  $-18^\circ$ ?

The sine wave with the phase angle  $\theta$  of  $72^\circ$  will reach its maximum value before the sine wave having a phase angle  $\theta$  of  $-18^\circ$ .

3. Given the following two sine wave equations:

$$E(t) = 8 \sin 20t + 78^\circ$$

$$E(t) = 40 \sin 20t + 43^\circ$$

Calculate the phase shift between these two sine waves, considering the first sine wave as the reference. Indicate also whether the second sine wave is lagging or leading the reference sine wave.

Phase shift =  $43^\circ - 78^\circ = -35^\circ$

The second sine wave is lagging the reference (first) sine wave.

4. When calculating the phase shift between two sine waves, which of the following parameters do the two sine waves need to have in common: phase angle, amplitude, frequency, or period? Why?

To calculate the phase shift between two sine waves, it is necessary for them to have the same frequency and period. Amplitude and phase angle have no effect on the duration of the sine wave cycle.

5. Consider two sine waves with the same frequency. They both have a period  $T$  of 50 ms. The second sine wave reaches its maximum positive value 8 ms after the first. Calculate the phase shift between the two sine waves, considering the first one as the reference.

Phase shift =  $\frac{d}{T} \cdot 360^\circ = \frac{8 \text{ ms}}{50 \text{ ms}} \cdot 360^\circ = 57.6^\circ$

Since the second sine wave is lagging the reference, the value of the phase shift is negative, thus:

Phase shift =  $-57.6^\circ$  or  $57.6^\circ$  lagging



## Bibliography

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Wildi, Theodore, *Electrical Machines, Drives, and Power Systems*, 6th ed., Upper Saddle River: Prentice Hall, 2005, ISBN 978-0131776913.